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Abstract. A recirculating accelerator accelerates the beam by passing through accelerating cavities multiple times. An FFAG recirculating accelerator uses a single arc to connect the linacs together, as opposed to multiple arcs for the different energies. For most scenarios using high-frequency RF, it is impractical to change the phase of the RF on each pass, at least for lower energy accelerators. Ideally, therefore, the FFAG arc will be isochronous, so that the particles come back to the same phase (on-crest) on each linac pass. However, it is not possible to make the FFAG arcs isochronous (compared to the RF period) over a large energy range. This paper demonstrates that one can nonetheless make an FFAG recirculating accelerator work. Given the arc's path length as a function of energy and the number of turns to accelerate for, one can find the minimum voltage (and corresponding initial conditions) required to accelerate a reference particle to the desired energy. I also briefly examine how the longitudinal acceptance varies with the number of turns that one accelerates.

LATTICE DESCRIPTION

For the purposes of this paper, a recirculating accelerator consists of an alternating sequence of identical linacs and arcs. The arcs are identical in the sense that each has the same path length as a function of energy. The linacs are identical in the sense that they all have the same voltage and the same phase. By "the same phase," I first mean that the phase of the RF does not change from one turn to the next. Second, if there are M linacs in the recirculating accelerator, the phase of one linac differs from that of the previous linac by $2\pi k/M$ for some fixed integer k. If one has an FFAG arc with a particular characteristic variation of path length with energy, one can cause the path length at one particular energy to give the same phase in the linac following the arc as in the linac preceding the arc by a combination of adding an additional length to the arc and varying the aforementioned k. Changing k allows one to keep the required changes to the dynamics of the arc lattice small.

There are two extremes in this design: one is the race-track design, where there are two long parallel linacs connected by arcs; the opposite extreme is a distributed RF system, where one has a sequence of short arcs with a single RF cavity between them. The racetrack design allows one to attempt to suppress dispersion in the linacs, eliminating longitudinal-transverse coupling. It is generally difficult to suppress dispersion over a large energy range, and in addition, the dispersion suppression may reduce the dynamic aperture of the system. The distributed RF system allows longitudinal-transverse coupling, but maintains a high degree of symmetry in the system, in principle giving a good dynamic aperture. The longitudinal-transverse coupling may not be so impor-
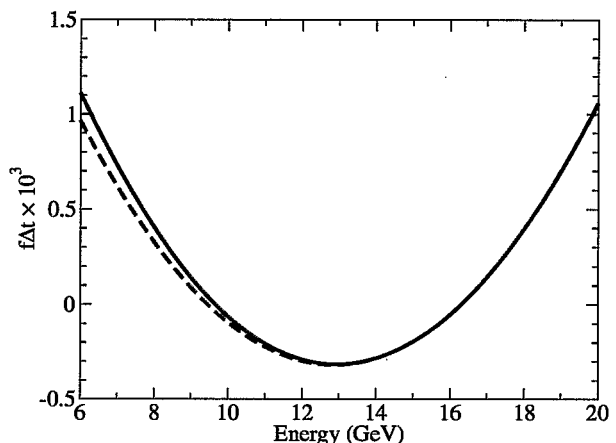

FIGURE 1. Path length as a function of energy for a single 6.5 m FFAG cell [1], calculated using COSY INFINITY [2]. Path length is given as a fraction of a 200 MHz RF period. The solid line is the exact path length variation, the dashed line is a quadratic approximation.

tant, however, since we are on-crest, and the energy gain does not vary so strongly with time-of-flight (that variation is what causes the longitudinal-transverse coupling).

It turns out that the path length in an FFAG arc is generally well approximated as a quadratic function of energy (see Fig. 1 for an example). One would expect that it is desirable to minimize the total variation in the path length over the desired energy range, and so one generally adjusts the lattice design to place the minimum of the parabola in the center of the energy range of the arc. Thus, the path length as a function of energy takes the form

$$\Delta T \left(\frac{2E - E_{\max} - E_{\min}}{E_{\max} - E_{\min}} \right)^2 - T_0. \qquad (1)$$

We expect ΔT to vary very little as T_0 is adjusted over a small range. Adjusting T_0 changes the energies for which the particle will see the same phase betweeen subsequent linacs.

Time-of-flight variation with energy is ignored in the linacs. For the purposes of this study, and considering the relatively large energies that these recirculating accelerators are designed for, it is a very good approximation to distribute any path length variation with energy in the linacs into the adjacent arcs.

EQUATIONS OF MOTION

The equations giving the energy and time-of-flight at the entrance and exit of the linacs are

$$E_{n+1} = E_n + V\cos(\omega\tau_n) \qquad (2)$$

$$\tau_{n+1} = \tau_n + \Delta T \left(\frac{2E_{n+1} - E_{max} - E_{min}}{E_{max} - E_{min}}\right)^2 - T_0. \qquad (3)$$

Here E_n is the energy after the nth pass through a linac and τ_n is the time-of-flight relative to the crest in the nth linac. It is useful to change coordinates to $x_n = \omega\tau_n$ and

$$p_n = \frac{2E - E_{max} - E_{min}}{E_{max} - E_{min}}. \qquad (4)$$

Then the equations become

$$p_{n+1} = p_n + v\cos x_n \qquad (5)$$

$$x_{n+1} = x_n + \Delta\phi\, p_{n+1}^2 - \phi_0 \qquad (6)$$

where $v = 2V/(E_{max} - E_{min})$, $\Delta\phi = \omega\Delta T$, and $\phi_0 = \omega T_0$.

Say we want to accelerate from E_{max} to E_{min} in N turns. Then $p_0 = -1$ and $p_N = 1$. The problem that we wish to solve is given these endpoint conditions, minimize v by varying x_0, the phase at which you enter the first linac, and ϕ_0. This solution will depend only on $\Delta\phi$ and N.

Note that this problem can be formulated for any voltage profile (e.g., something different from $\cos\phi$) as well as a different relationship of time-of-flight to energy. Changing the voltage profile will only change the results quantitatively, not qualitatively [3]. Changing the relationship of time-of-flight to energy will be discussed later.

Continuous Approximation

It is useful to make a continuous approximation to Eqs. (5-6) [3]. The continuous approximation indicates that for $N\Delta\phi$ large, $v \to \Delta\phi/12$, $\phi_0 \to \Delta\phi/4$, and $x_0 \to -\pi/2$. The reason that these results are interesting is that they give a good approximation to what will occur in the

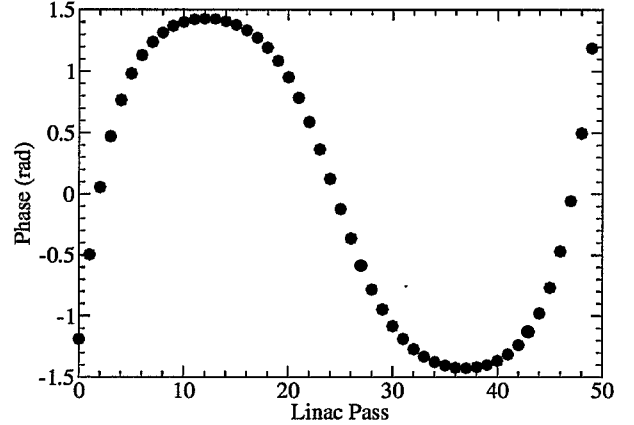

FIGURE 2. Phase as a function of linac pass when $\Delta\phi = 1$ and $N = 50$.

discrete case. In particular, since the continuous approximation has a solution for all N, we might expect that the discrete system does as well. In the case of the distributed RF system, $\Delta\phi$ is small and N is very large, and the discrete equations are in fact a very good approximation to the continuous ones. For a racetrack system, however, it is far from clear that the approximation is good. Since $\Delta\phi$ is relatively large, the change of x in one step can be large, making it questionable whether the contuous approximation is really very good. However, we will subsequently see that for a large number of turns, a large fraction of the steps occur at points where the change in x is small, and x_n is large and therefore the change in p is also small. The continuous approximation thus turns out to give the correct results for large numbers of turns both qualitatively and nearly quantitatively as well.

EXAMPLE

We now find the minimum-v solution of Eqs. (5-6) for $\Delta\phi = 1$. This is a relatively large phase swing: remember that the phase errors accumulate, and so after only 4 steps with this phase error, one would certainly be decelerating. This example is more appropriate for a racetrack configuration: the phase swing per arc would be orders of magnitude smaller for a distributed arc system (but there would be correspondingly more linac passes required).

If one makes 50 linac passes for this system and performs the aforementioned optimization, the phase as a function of turn number is shown in Fig. 2. Note that the reference particle crosses the crest three times; this is related to the parabolic shape of the path length. The particle spends most of its time at the two turning points in phase (turns 8-18 and 32-42); these are the points where the path length error is near zero. Due to the large phase at this point, the particle is not gaining very much energy, and so remains at the point where the path length is near
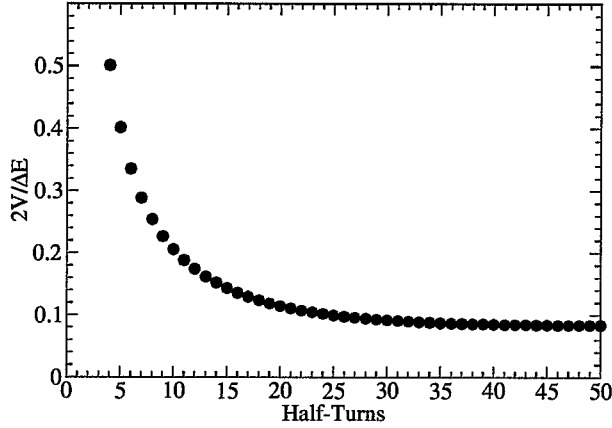
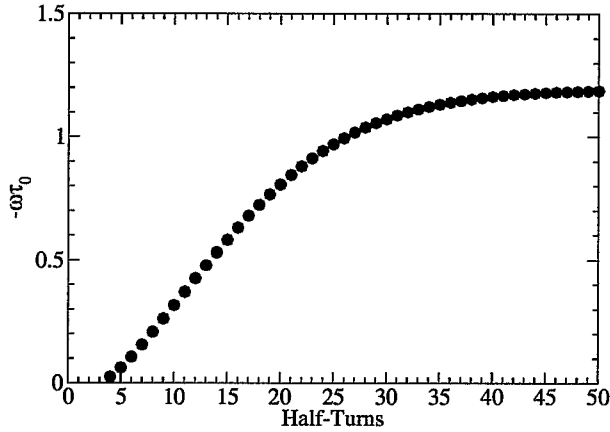**FIGURE 3.** Linac voltage as a function of the number of linac passes.

FIGURE 4. Initial phase as a function of the number of linac passes.

FIGURE 5. Initial phase space that is accepted in a 8 linac pass system. Each color represents a band of ±0.01 in p_N.

FIGURE 6. Initial phase space that is accepted in a 24 linac pass system.

zero for a long time. This is the mechanism by which the particle can spend an arbitrary number of turns in this system.

Figure 3 shows the voltage as a function of the number of linac passes, and Fig. 4 shows x_0. For large numbers of turns, these (including ϕ_0, not shown) do appear to be approching the large N limits given above. Note that while it is not clear that $x_0 \rightarrow -\pi/2$ in Fig. 4, from Fig. 2, one can see that the maximum phase swing is slightly larger than $-x_0$. The fact that $\Delta\phi$ is large causes this difference from the continuous approximation. It is really the regime near the turning points in phase that approaches the continuous approximation. It turns out that the voltage limit as $N \rightarrow \infty$ is very slightly less than what is found in the continuous approximation, and the difference is also due to the the the finite $\Delta\phi$.

Finally, Figs. 5 and 6 demonstrate how the acceptance varies with the number of turns. A smaller number of turns seems to caus a large region of phase space to be accepted; what one sees at 8 linac passes is very close to
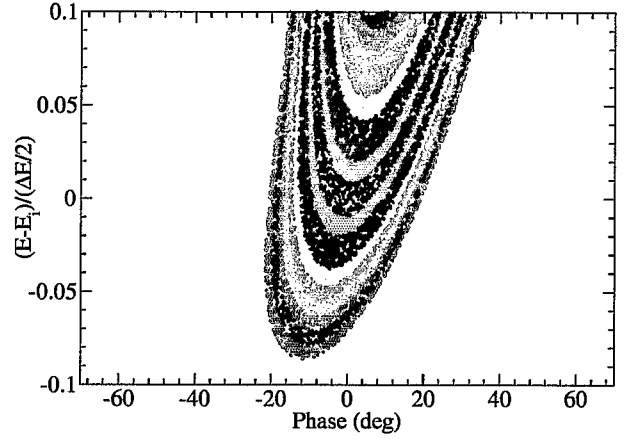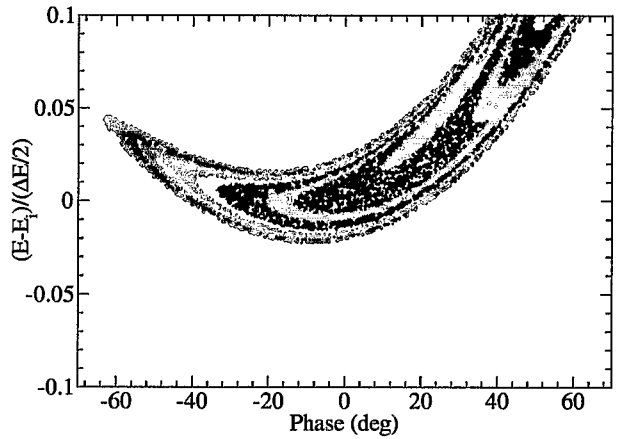
what one expects from on-crest acceleration. For a large number of linac passes (24 here), one notes that most everything that is accepted ends up within a small band of the reference momentum at the final energy. In fact, while it appears that there is a smaller total acceptance for more turns, the phase space area ending up within a small energy band at the end is mmuch larger for more turns. The analysis of acceptance is still very preliminary, and must be studied more thoroughly.

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2. K. Makino and M. Berz. COSY INFINITY version 8. *Nuclear Instruments and Methods*, A427, pp. 338–343 (1999).
3. J. Scott Berg. Proceedings of Snowmass 2001.